COMPARISON OF M/M/1 & M/G/1 MODEL USING REAL TIME TRAFFIC-FLOW CONGESTION COSTS

Karthika.R
Research Scholar,
Nirmala College for women,
Coimbatore, Tamil Nadu, India.

Sahaya Sudha.A
Assistant Professor,
Nirmala College for women,
Coimbatore, Tamil Nadu, India.

Abstract- This Paper provides useful information about a queueing theory approach to congestion costs. We Develop some analytic queueing models like M/M/1 and M/G/1 based on traffic counts and we model the behavior of traffic flows as a function of some of the most relevant determinants. These models pave the way towards our research objective. The illustration of statistical data of real time traffic flow and a comparison of M/M/1 and M/G/1 model have been furnished using those results. In every respect this paper will definitely be the proper guidelines to the people in connection with traffic flow, time saving factor and pollution problem.

Keywords---- Uninterrupted Traffic, Congestion costs, Traffic density, real - Time Traffic flow, Traffic Intensity.

1. INTRODUCTION:

In the public sector, accurate modeling of operations and logistics functions is a necessary precondition to effective operational planning and control for a society as a whole. Due to increased ownership of cars, changes in the production system, increased flexibility of the working population, etc, the demand for transport has increased exponentially. This combination of increased inflow of traffic on the existing road network together with a stabilization in investments of new roads, results in an inevitable increase of congestion. Congestion leads to an increase in travel time, decreasing flow, higher fuel consumption, negative environmental effects etc. Efficiently turning demand and supply will lead to a better use of the capacity and better control of traffic demand. This solution can be achieved by a temporarily and local intervention on supply and demand. Traffic will be modeled using mathematical models based on queueing theory. Queues occur whenever instantaneous demand exceeds the capacity to provide a service. Queueing theory involves the mathematical study of these waiting lines. Using a large number of alternative mathematical models, queueing theory provides various characteristics of the waiting line, like waiting time length of the queue. Vandaele, Van Woensel and Verbruggen(2000) ,Van Woensel crelen and Vandaele (2001) and Heidemann.,D (1991,1994) showed that queueing models[4,5,9] can be a useful alternative for modeling uninterrupted traffic flows. In this paper we deal with estimating marginal congestion costs for uninterrupted traffic flows, such as delays caused by congestion on major roads in the Coimbatore city.

Traffic management has become very essential in our times where the number of vehicles in the metros are near the existing road capacity or at some places or at some places even beyond. India has witnessed this surge only in the past decade or so, and therefore, attention to traffic management has increased recently. In Delhi and Mumbai, Metro rail services have been introduced, and at many places a Bus rapid transit system has been initiated , but however many roads we may build, We will always fall short of the space needed to accommodate the ever increasing traffic.

2. PRELIMINARIES:

2.1 TRAFFIC FLOW:

Traffic flow can be divided into two primary types [9]

2.1.1 UNINTERRUPTED TRAFFIC FLOW:

The first type, uninterrupted flow, is defined as all the flows regulated by vehicle-vehicle interactions and interactions between vehicles and the roadway.

For example:- Vehicles travelling on a highway in the Coimbatore city are participating in uninterrupted flows.

2.1.2 INTERRUPTED TRAFFIC FLOW:

Interrupted flow is regulated by an external means , such as signal .

2.2 TRAFFIC CONGESTION COST:

Traffic congestion costs consist of incremental delay, driver stress, Vehicle costs , crash risk and pollution resulting from interference between vehicles in the traffic stream, particularly as a road system approaches its capacity.

2.3 TRANSPORT NETWORK:

It is typically a network of roads , streets , pipes, power tubes , power lines or nearly any structure which permits either vehicular movement flow of some commodity.

- A transport network is used for transport network analysis to determine the flow of vehicles.
- Transportation networks are used to model the flow of commodity and traffic.[1,3]

2.4 TERMS AND CONCEPTS:

- Traffic congestion can be recurrent (occurring regularly on a daily , weekly or annual cycle , making it easier to manage) or non- current (due to accidents , special events or road closures)
- Capacity refers to the number of people or vehicles that could be accommodated. Load factor refers to the position of capacity actually used.
- A queue is a line of waiting vehicles.
- A platoon is group of vehicles moving together (such as after traffic signals turn green) [2,7]

Following table 1 gives an overview of the parameters used:

PARAMETER	DESCRIPTION
λ	Arrival rate (veh / hr)
μ	Service rate (veh / hr)
ρ	Traffic Intensity $(\frac{\lambda}{\mu})$

E	Expected Frequency
O	Observed Frequency
L_s	Average number of vehicles in the system
L_q	Average number of vehicles waiting in the queue
W_s	Average waiting time of vehicle in the system
W_q	Average time spent by a vehicle in the signal point
σ	Standard deviation

Table-1

2.5 M / M / 1 MODEL:

For the M / M / 1 queueing model , the inter-arrival times are exponentially distributed (The arrival process follows a poisson process) with expected inter-arrival time equal to $1/\lambda$ (with λ equal to the product of the traffic density E and the nominal speed SN).[6,7]

The service time is also exponentially distributed with expected service time $1/\mu$ (The service rate follows a poisson distribution) . Here the service time denotes the time needed for a vehicle to pass one road segment , with expected service time μ).

2.6 M / G / 1 MODEL:

As in the M / M / 1 model inter-arrival times follow an exponential distribution with expected inter-arrival $1/\lambda$, λ being the product of traffic density and nominal speed . The service time is generally distributed with an expected service time of $1/\mu$ and a standard deviation of σ.[7]

2.7 χ^2 - DISTRIBUTION:

The χ^2 test is one of the simplest and most widely used non – parametric tests in statistical work. The χ^2 test was first used by Karl Pearson in the year 1900 .The quantities χ^2 describes the magnitude of the discrepancy between theory and observation[8]. It is defined as,

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

2.8 POISSON PROCESS:

$P_n(t)$ is given by the poisson law ;

$$P_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}, \quad n = 0, 1, 2, 3\ldots\ldots\ldots\ldots$$

In general we know that,

Poisson distribution is a limiting case of the binomial distribution.

Hence the probability mass function of a random variable . 'X' which follows Poisson distribution is given by[8],

$$P(x_i)=P(X=x)= \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!}, & x = 0,1,2\ldots\ldots\ldots\ldots\ldots\infty \\ 0, & \text{otherwise} \end{cases}$$

3. A QUEUEING APPROACH:

In the queueing approach to traffic flow analysis, roads are subdivided into segments, with length equal to the minimal space needed by one vehicle on that particular road (figure-1). [6,9]

Figure 1. Queueing representation of traffic flows

4. CASE STUDY:

The statistical analysis of timings of the traffic signal and the number of vehicles which are about to cross the signal clearly shows that if this type of procedure is continued either the time element or the vehicle movement may not be truly coincide further due to population explosion and too much usage of two wheeler and four wheelers during the busy hours create creates a chaotic situation . Due to this either the vehicle users or pedestrians are not properly helped to arrive a smooth and neat solution to make this confused situation to a normal but at the same time a speedy manner this finding will be highly useful and helpful .

Inter-arrival time	Arrival rate	Inter-departure time	Service rate
8:00:01 - 8:01:00	32	8:01:00 - 8:01:30	25
8:01:31 - 8:02:30	51	8:02:31 - 8:03:00	23
8:03:01 - 8:04:00	38	8:04:01 - 8:04:30	20
8:04:31 - 8:05:30	49	8:05:31 - 8:06:00	21
8:06:01 - 8:07:00	33	8:07:01 - 8:07:30	26
8:07:31 - 8:08:30	50	8:08:31 - 8:09:00	28
8:09:01 - 8:10:00	49	8:10:01 - 8:10:30	21
8:10:31 - 8:11:30	55	8:11:31 - 8:12:00	26
8:12:01 - 8:13:00	48	8:13:01 - 8:13:30	35
8:13:31 - 8:14:30	42	8:14:31 - 8:15:00	33
8:15:01 - 8:16:00	47	8:16:01 - 8:16:30	36
8:16:31 - 8:17:30	52	8:17:31 - 8:18:00	30
8:18:01 - 8:19:00	39	8:19:01 - 8:19:30	27
8:19:31 - 8:20:30	43	8:20:31 - 8:21:00	31
8:21:01 - 8:22:00	37	8:22:01 - 8:22:30	26
8:22:31 - 8:23:30	36	8:23:31 - 8:24:00	23
8:24:01 - 8:25:00	41	8:25:01 - 8:25:30	29
8:25:31 - 8:26:30	38	8:26:31 - 8:27:00	33
8:27:01 - 8:28:00	30	8:28:01 - 8:28:30	29
8:28:31 - 8:29:30	34	8:29:31 - 8:30:00	27

A neat statistical records pertaining to time element , departure of vehicles time for the pedestrians to cross the roads have been categorically mentioned below to have a clear idea regarding uninterrupted traffic flows. All the materials relevant to the above has been collected and recorded from traffic signal of Ramanathapuram road , Coimbatore which is the main junction of the southern part of the Coimbatore district.

Table: 2 Real-time traffic flow data:

4.1 ALGORITHMIC APPROACH:

The steps required to determine the value of χ^2 are as follows: [8]

Step -1:

Calculate the Expected frequencies using the formula $E = P(x_i) * N$.

Step -2:

Take the difference between observed and expected frequencies and obtain the squares of these difference (i.e) Obtain the values of $(O-E)^2$.

Step -3:

Divide the values of $(O-E)^2$ obtained in step 2 by the respective expected frequency and obtain the total $[\sum \frac{(O-E)^2}{E}]$. This gives the values of χ^2 which can range from 0 to ∞.

Step -4:

The calculated value of χ^2 is compared with the table value of χ^2 for given degrees of freedom at a certain specified level of significance .(In general it is assumed to 5 % level).

Case – 1:

If at the stated level (generally 5 % level of significance is selected), If the calculated value of χ^2 is less than the table value of χ^2 then the difference between theory and observations is considered to be significant and the hypothesis is said to be accepted.

Case – 2:

If the calculated value of χ^2 is more than the table value, the difference between theory and observation is not considered as significant and the hypothesis is said to be rejected

Fitting a Poisson distribution of the arrival rate λ:

Arrival rate $\lambda = \frac{\Sigma fx}{\Sigma f} = \frac{844}{20} = 42.2$ **per min**

$$P(x_i) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-42.2}(42.2)^x}{x!}$$

$$P(30) = \frac{e^{-42.2}(42.2)^{30}}{30!} = 0.010197$$

$$E_1 = 0.010197*20 = 0.2039$$

Similarly the remaining values are calculated as in above table.

$$\sum\frac{(O-E)^2}{E} = 14.38919$$

$$\Rightarrow \chi^2 = \sum\frac{(O-E)^2}{E} = 14.3892$$

Here the degrees of freedom is

$$\gamma = 20 – 1 = 19$$

hence at 5 % level of significance the table value is 31

$$\text{(i.e.)} \quad \chi^2_{0.05} = 31$$

$$\therefore \chi^2 = 14.38919 < 31$$

Hence the difference between theory and observations is significant at 5 % level of significance.

∴ Arrival rate follows Poisson distribution.

Fitting a Poisson distribution of the service rate μ:

Service rate

$\mu = \frac{\Sigma fx}{\Sigma f} = \frac{555}{20} =$ **27.8 in 30secs.**

$$P(x_i) = \frac{e^{-\mu}\mu^x}{x!} = \frac{e^{-27.8}(27.8)^x}{x!}$$

$$P(20) = \frac{e^{-27.8}(27.8)^{20}}{20!} = 0.02638$$

$$E = P(x_i) * N$$

$$E_1 = 0.02638*20 = \mathbf{0.5277}$$

Proceeding like this remaining values are calculated as in the above table.

$$\therefore \sum\frac{(O-E)^2}{E} = 7.10158$$

$$\Rightarrow \chi^2 = \sum\frac{(O-E)^2}{E} = 7.10158$$

The degrees of freedom is

$$\gamma = 20 - 1 = 19$$

The table value of $\chi^2_{0.05} = 31$

At 5% level of significance , the calculated value is 7.10158 < 31

Hence the difference between theory and observations are significant at 5 % level of significance.

∴ Service rate follows Poisson distribution.

∴ Arrival rate $\lambda = 42.2$ in 60 sec

Arrival rate λ = **3.52 in 5 sec**

Service rate μ = 55.6 in 60 sec

Service rate μ = **4.63 in 5 sec**

$\therefore$ Traffic Intensity $\rho = \frac{\lambda}{\mu} =$ **0.759 < 1**

M / M / 1 MODEL:

$$L_s = \frac{\lambda}{\mu-\lambda} = \frac{3.52}{4.63-3.52} = 3.171$$

$$W_s = \frac{1}{\mu-\lambda} = \frac{1}{4.63-3.52} = 0.9009$$

$$L_q = \frac{\lambda^2}{\mu-\lambda} = \frac{(3.52)^2}{4.63-3.52} = 11.16$$

=**11vehicles** approximately

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{3.52}{4.63(4.63-3.52)} = \mathbf{0.6849}$$

If T represents the service time for a vehicle then variance of T is given by,

$$Var(T) = \sigma^2 = \frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f}\right)^2$$

$$\therefore \sigma = \sqrt{\left(\frac{15825}{20} - \left(\frac{555}{20}\right)^2\right)}$$

$$= \mathbf{4.6029}$$

$\therefore \sigma = 4.6029$ in 30 sec

= 0.76715 in 5 sec

$\therefore \sigma^2 =$ **0.5885 in 5 sec**

M / G/ 1 MODEL:

Pollaczek-Khinchine Formula :- (PK – Formula)

$$L_s = \rho + \frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)}$$

$$=0.75 + \frac{(3.52)^2(0.5885) + (0.759)^2}{2(1-0.759)}$$

$$= \mathbf{17.0823}$$

$L_q = L_s - \frac{\lambda}{\mu} = 17.0823 - 0.759 =$ **16.3233**

$$W_q = \frac{L_q}{\lambda} = \frac{16.3233}{3.52} = \mathbf{4.6373}$$

$$W_s = \frac{L_s}{\mu} = \frac{17.0823}{4.63} = \mathbf{3.689}.$$

5. DISCUSSIONS:

A comparative study of M/M/1 and M/G/1 models are made using the real time traffic flow and the average number of vehicles waiting time in the system and in signal point are calculated and furnished as follows,

Little's formula	**M/M/1 Model (5 sec)**	**M/G/1 Model (5 sec)**
L_s	**3 vehicles**	**17 vehicles**
W_s	**1 vehicle**	**4 vehicles**
L_q	**11 vehicles**	**16 vehicles**
W_q	**1 vehicle**	**5 vehicles**

Table:-3 Comparative study of models

With lots of information and details collected from authentic sources , a new and at the same time pragmatic suggestion have been brought out to ease the long pending issue of traffic flow. Hence the above statistical table has been practically proved that M/M/1 model is much better when compared to M/G/1 model.

6. CONCLUSION:

A bright future for the benefit of the society depends upon the present findings with the aim of suggesting best pragmatic approaches. Traffic problem is the prime factor for many disturbances which all types of persons are worried and facing it with lot of patience. Due to this particular problem, important duties are not able to be fulfilled at the appropriate time.***"Time and Tide waits for no man".*** The winds and waves are always on the side of the ablest navigators. For every problem, there is a solution. To find out useful and correct solution, a thorough research is necessary, for which technique is the basic one. Uninterrupted traffic flows is the main target of my paper.

Traffic is the main artery of life. If there is not a free flow of traffic, life will be miserable. The main ingredient of humanism is giving our helping hand to society to relieve their inexpressible pain and agony that is facing every day. The real meaning of civilization is to have useful means and ways to wipe the problems that society is facing.

This study very clearly pictured various factors responsible for traffic congestion at a particular place. If this fault is not neatly solved, like chain reaction , it will lead to total failure of traffic route. To avert this, construction of flyovers and subways will ease the problem. To a great extent due to this pollution will be curtailed to a certain extent.

My choice of the solution of this topic is mainly to help the traffic flow without any congestion, thereby I hope my findings will be useful to the future set of traffic rules and regulations.

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